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INDIVIDUAL PATHWAYS IN THE DEVELOPMENT OF STUDENTS' CONCEPTIONS OF PATTERNS OF CHANCE

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An essential property for distinguishing random from haphazard events is the existence of patterns in the long term. Its inclusion into the individual repertoire of conceptions counts as a prerequisite to developing adequate conceptions of chance and probability. This paper exemplifies results from a teaching experiment designed to investigate 11 to 13 year-old students' individual pathways of constructing, enriching and refining their conceptions of patterns of chance

Students' individual conceptions of chance and probability have often been investigated empirically. The construction of conceptions that match the underlying stochastic theory (shortly called *intended* or *mathematically-appropriate* conceptions) seems to be a major challenge for stochastic education (e.g. Shaughnessy, 1992) and is deeply influenced by students' initial everyday conceptions (Fishbein, 1975; Konold, 1989). While early conceptualisations of these initial conceptions labelled them as mis-conceptions (e.g. overview in Shaughnessy, 1992), stochastic education researchers with a constructivist background have taken them seriously as starting points for individual learning processes (e.g. Konold, 1989; Pratt & Noss, 2002) and studied their development. In this tradition, this paper aims at contributing to a deeper understanding of students' individual *pathways of constructing, enriching and refining their conceptions of patterns of chance* as observed in design experiments.[1]

THEORETICAL BACKGROUND

Context-differentiated activation of constructs as an aim for processes of horizontal conceptual change

The relevance of individual initial (mis-)conceptions for the construction of conceptions has been explained in constructivist terms: individual, active constructions of mental structures always build upon the existing prior mental structures by accommodation to experiences with new phenomena, while the initial structures serve as "both a filter and a catalyst to the acquisition of new ideas" (Confrey, 1990, p. 21). According to the conceptual change approach (Posner et al., 1982; first applied to probability by Konold, 1989), learning thus has to be conceptualised as "re-learning, since prior conceptions and scientific conceptions are often opposed to each other in central aspects" (Duit & von Rhöneck, 1996, p. 158). For many years, conceptual change approaches have (implicitly or explicitly) guided the design of learning situations by providing means to

[1] The design experiments are embedded in the long-term project KOSIMA that conducts design research for a complete middle school curriculum (cf. Hußmann, Leuders, Barzel, & Prediger, 2011).

overcome initial conceptions and develop them into intended mathematically appropriate conceptions. These means concern, for example, the relevance of concrete experiences, the confrontation of predictions with real outcomes and the generation of cognitive conflicts (see Posner et al., 1982). However, the far reaching aim of "overcoming" individual prior conceptions in mathematics classrooms that guided early views on conceptual change is not universally applicable. Empirical studies show that it is not always realisable, as individual conceptions often continue to exist next to the new conceptions and are activated situatively (cf. Tyson et al., 1997; for probability e.g. Shaughnessy, 1992; Konold, 1989). Rather than a substitution of initial conceptions, the more adequate aim is the *shift of contexts* in which initial and intended conceptions are to be activated. "Successful students learn to utilize different conceptions in appropriate contexts." (Tyson et al., 1997, p. 402). Pratt & Noss (2002) emphasise changes in priority between initial and intended conceptions as one pathway of a conceptual change.

Prediger (2008) called this modified perspective on conceptual change with persisting co-existence of initial and intended conceptions a *horizontal view*; in contrast to the *vertical* view on conceptual change, which aims at overcoming initial conceptions. The horizontal view considers students' initial conceptions as legitimate ideas that can persist if they are weaved into a new framework (similar to Abrahamson & Wilensky, 2007) and can be refined by knowledge of their context-specific scope of validity. Thus, the question guiding the design and analysis of a learning situation for facilitating *horizontal* conceptual change transforms into the following: *How can a learning situation support the extension of individual repertoires of conceptions* (constructing and enriching)*, and how can learners be enabled to choose adequate conceptions in varying contexts* (refining and generalising)*?*

For terminological clarification, we mention that in line with the conceptual change approach, the notion 'conception' here refers to all subjective mental structures used by learners to explain their experiences. Conceptions may range on different epistemological levels of complexity from concepts, intuitive rules up to local theories that connect different concepts (Gropengießer, 2001, p.30ff.) and can vary in the degree to which they match the underlying mathematical theory. Although the conceptual change approach is suitable to describe the macro-structures in individual pathways of development of conceptions (see for example Prediger & Rolka, 2009), the fine-grained analysis of micro-structures in the processes of constructing, enriching and refining conceptions require a further operationalization on the micro-level (similarly in diSessa, 1993; Pratt & Noss, 2002). For this purpose, we adopted the notion 'construct' as the smallest empirically-identifiable unit of conceptions from Schwarz et al. (2009) and their methodology of reconstructing them by means of three observable *epistemic actions*: Conceptions are seen as webbings of constructs. An epistemic action of *constructing* is defined as (re-)creating a new knowledge construct by *building with* existing ones. This is identified when a construct is first verbalised or shown by action in the analysed learning situation (although sometimes being constructed before the observed situation). Previous constructs can be *recognized* as relevant for a specific context and used for *building-with* actions in order to achieve a localized goal.

Due to our horizontal view, two major adaptions of the notions were necessary: 1. As we consider idiosyncratic conceptions to be legitimate building blocks, we extended the normatively-guided focus from mathematically (partially) correct constructs (Ron et al., 2010) to all individual constructs, being in line with mathematical conceptions or not. 2. Our descriptions of horizontal learning pathways are mainly focused on the epistemic actions of constructing and required the distinction of two special cases of constructing, namely enriching and refining. A construct is identified to be *enriched*, when a complementary construct is put into relation to it which means there are connections to other constructs identifiable. A construct is said to be *refined*, when it is enriched by conditions of applicability; in our study mostly as narrowing the range of applicable situations from a broad initial one. In other situations, initial constructs are *generalized* and transferred to new contexts (as reconstructed e.g. by Pratt & Noss, 2010, p. 94).

Conceptions of patterns and deviations distinguishing long-term and short-term contexts as precondition for context-adequate choices

The existence of patterns in long series of chance experiments can be identified as a crucial insight for developing adequate conceptions of chance and probability (Prediger, 2008). This focus is strengthened by Moore's definition of random as "phenomena having uncertain individual outcomes but a regular pattern of outcomes in many repetitions" (Moore, 1990, p. 97). This includes the important distinction between short-term and long-term contexts which is central since Konold (1989) described many people's "different understanding of the *goal* in reasoning under uncertainty" (p.61, emphasis added) as an important source of deviant conceptions. Whereas probabilistic conceptions only apply to long-term contexts, many people intend to predict single outcomes of chance experiments in a short-term perspective (Konold, 1989). Deviant conceptions — like betting on numbers that have a specific significance such as birthdays — can be experienced as unsuccessful in long-term contexts, but they prove just as (un)suitable — for single outcomes — as the intended probabilistic conceptions. Therefore, the well-known empirical law of large numbers is crucial for horizontal conceptual change since it explains why one can adopt probabilistic conceptions in a successful way (in long-term contexts), although randomness cannot be predicted for single outcomes (the short-term context). The empirical law of large numbers explains the *sense* and *preconditions*, but also the *limits* of probabilistic considerations and offers thus the conceptual base for a context-adequate choice of conceptions.

Borovcnik (2006) emphasised that the learning process while experimenting with dice etc. is hindered by the fact that chance, and therefore the produced data does not only have *patterns*, but also many *deviations*. That is why students have to include these experiences into their conceptions. Therefore, developing context-adequate probabilistic conceptions does not only include the important shift of attention from short-term contexts to long-term contexts (cf. Pratt & Johnston-Wilder, 2007), but also the construction of conditions *when* regularities are visible: whereas patterns are visible in sufficiently long series of outcomes, they can be disturbed by many outliers in short series, and single outcomes might not conform to an expected pattern at all (see Table 1). In this paper, we describe a case of successful development while constructing, enriching and refining constructs of patterns and their deviations in relation to the context.

DESIGN OF THE TEACHING EXPERIMENTS

The learning situation based on 'Betting King'

To facilitate the differentiation between short-term and long-term contexts in the sense of a horizontal view of conceptual change, a learning situation for 11 to 13 year old students has been designed by Prediger & Hußmann (2012) to provide opportunities for experiences with the empirical law of large numbers. The core element of the learning situation is the board game "Betting King" (Fig. 1), which challenges students to bet on one of four coloured animals in a race. Betting activities refer to making predictions which animal will be the fastest and on which position each animal will end up. The four coloured animals are powered by throws of a coloured 20-sided die (red ant: 7, green frog: 5, yellow snail: 5, blue hedgehog: 3), so that the red ant is theoretically the fastest with a chance of 7/20. Most children quickly notice the red ant to be a *good bet.* Soon, they activate a fruitful ordinal conception of chance, relating the expected order of animals to the number of coloured faces on the die. In this way, the students' initial resources to link the empirical pattern to the colour distribution are taken into account. Beyond that, the learning situation aims to refine these initial conceptions into an understanding of when this pattern can be predicted more confidently according to the long-term or short-term context. For this purpose, the context attribute "total of throws" is materialised in the game by a STOP sign for the throw counter. By setting the STOP sign for each game, students can deliberately define the total of throws between 1 and 40 for the board game, and between 1 and 10000 throws for the computer simulation (Fig. 2). In order to lead students from unsystematically playing the game into systematically investigating the situation, protocol sheets guide the collection of game result data for various predefined throw counts (1, 10, 100 and 1000, later 2000). For refining constructs by the conditions of their applicability, it is important to become aware of the role of the total number of throws.

Fig.1 The Game 'Betting King'

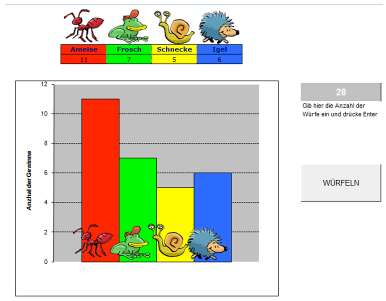

Fig. 2 Screen of corresponding computer simulation

	Short-Term Context: Single games with small total of throws	**Long-Term Context 1:** Series of games with small total of throws	**Long-Term Context II:** Series of games with large total of throws
Pattern	**S-pattern** non-existent	**L1-pattern** red ant mostly wins, frog & snail are in similar positions, hedgehog loses mostly	**L2-pattern** red ant always wins, frog & snail are second, hedgehog last
Quality of prediction	**S-predictability** difficult to bet, but red ant is still the best	**L1-predictability** red ant is a good bet, but not a secure bet	**L2-predictability** red ant is a good and secure bet
Relevance of disturbance	**S-disturbance** some single outcomes completely differ from any expected pattern	**L1-deviation** pattern difficult to see due to lot of disturbances	**L2-deviation** pattern strongly visible, still some disturbance
Explanation of appearing pattern	no adequate explanation for the outcome itself	e.g. L1-theoretical explanation L1-empirical explanation	e.g. L2-theoretical explanation L2-empirical explanation L2-law of large numbers

Table 1: Intended probabilistic constructs on patterns of chance in Betting King

From a probabilistic point of view, the distinction between different long-term contexts and a short-term context is crucial as was elaborated above. But whereas this distinction oriented our data-guided systematisation of intended constructs in Table 1, students first have to construct this distinction between contexts by themselves. In our learning situation, this construction of differences is facilitated by the following leading question: "Apparently, the red ant is a *good bet*. But when is this bet not only the best bet but also a *mostly secure bet*?" The question challenges students to enrich their conceptions by context-differentiated constructs of predictability which are linked to individual constructs of relevance of disturbance, but in many different ways, as we learned in the ongoing data analysis.

Research Questions and Design of the Study

Following the paradigm of design research (Gravemeijer & Cobb, 2006), the learning situation was tested and improved cyclically over three courses of evaluation in six classes (grade 5 and 6, students aged 11 to 13). The empirical analysis of classroom learning processes in Prediger & Rolka (2009) showed that most students could indeed find better and more secure betting strategies and learned to differentiate between long-term and short-term contexts. However, for gaining a deeper understanding on the detailed *processes* of the development of conceptions, classroom data was too incomplete. For that reason, a further series of teaching experiments was conducted in a laboratory situation by the second author of this paper.

The teaching experiments (following Gravemeijer & Cobb, 2006) based on the presented learning situation were conducted in a series of game interviews with ten couples of students of grade 6 (age 11-13) in a German comprehensive secondary school. The semi-structured interviews of 4x 45-90 minutes were guided by an intervention manual that defined the role of the interview with the attitude of giving as little help as possible but also to provide guidance in situations that were crucial for the continuation of the interview sessions. Each session was videotaped and transcribed in detail for the analysis. The data corpus also included the record of the computer screen and written products.

Though the underlying research interest of the ongoing analysis addresses a range of different questions concerning a more detailed description of the processes of conceptual change, this article focuses on the following questions:

- How are students constructing, enriching and refining constructs for patterns, deviations of patterns and predictability in relation to short- and long-term contexts?
- Which constructs do students use for explaining these patterns?

THE CASE OF RAMONA AND SARAH – FIRST RESULTS

The case of Ramona and Sarah exemplifies how relating and enriching different constructs can provide students with a tool to make sense of the different observations of patterns of chance in relation the specific short-term or long-term context. This case was chosen as the girls show a broad range of constructs and are highly able to verbalize their ideas. Due to limited space, comparisons with other couples are restricted to the concluding remarks.

Episode 1: Refining by differentiating the L1-context from S-context

When introduced to the learning environment, Ramona and Sarah are eager to find a strategy to win as often as possible. For this reason, they keep looking for patterns in single throws of the die or in results of games. Episode 1 below starts after 15 minutes of playing. All four games so far with totals of throws between 25 and 37 have been won by the red ant, with the first game tied with the green frog. Having the outcome of the fourth game on the board as documented in Fig. 1 (red ant on 11, frog on 7, snail on 5, blue hedgehog on 6), the students express their ideas on the found patterns.

422	Ramona	*(points to red ant on the board)* This one is the fastest. Then, the hedgehog should come, then [green] frog, then [yellow] snail *(points to animals)*
423	Sarah	Why?
424	Ramona	I don't know, because the- the ant has won almost every time so far.

As a first construct, Ramona describes in line 424 the pattern that the red ant wins more often (L1-pattern winning ant) and relates this to her empirical observation (L1- empirical explanation). She apparently refers to the series of four games by expressing "almost every time so far" in 424. In the (not printed) turns following the above episode, Sarah tries to find an explanation for the empirical pattern and comes up with the idea that not all faces of the die are equal, which prompts Ramona to count. After counting twice, they find the correct colour distribution of 7,5,5,3.

481	Interviewer	Now you have counted [all colours on the die]. What does that mean?

482a	Sarah	That red, well, more- well, that red wins actually, because it has more and then you get it more often, when you throw the die. And then green and yellow, because they-
482b	Sarah	Well, you two- That is why they are again so- Green and yellow (*points on yellow snail and then blue hedgehog on the board*)
482c	Sarah	Eh, green and yellow (*points to yellow snail and then green frog, then to both simultaneously*) are sometimes far apart, but.
483	Ramona	Blue has good chances, too, because-
484	Sarah	Yes.
485	Ramona	You also have- blue has sometimes a lot of luck and then it gets the three faces sometimes very often.
486	Sarah	You see it here *(points on the board to snail and blue hedgehog)*.

In 482a, Sarah enriches the pattern-construct that was so far only empirically explained with an additional theoretical explanation of the colour distribution (L1-theoretical explanation). While the observation and also the empirical explanation of the pattern of the red ant as best animal come from a *series* of games (with totals between 25 and 37, L1-context), she switches in 482b to the *single* result of the game that is still displayed in front of her (see Fig.1) and tries to transfer the L1-pattern to the single game. By pointing to the board, she is possibly trying to demonstrate the theoretically expected pattern, but her use of half sentences and her pointing to the wrong animals in 482b seem to indicate that she is experiencing a conflict between the deviant S-pattern and the expected L1-pattern. In this moment, the constructed L1-pattern is possibly already starting to get refined implicitly as Sarah experiences a problem in its scope of applicability for the single short game. In 482c, she corrects herself by pointing to green frog and yellow snail, but seems not to be describing a pattern anymore, as she uses the term "sometimes" (S-deviation). Sarah seemingly does not solve the problem between L1-pattern and the deviant S-pattern here, as she ends her sentence with a "but" in 482c, even though she is not interrupted.

Episode 2: Constructing luck as S-explanation for deviation

Ramona expresses in 483 to 485 a new construct that had not been mentioned before. She explains this situation that differs from the L1-pattern by the "luck" that the blue hedgehog must have had (S-explanation for the deviation). Keeping the term "sometimes", she is seemingly still speaking about single outcomes as opposed to a series. Sarah concurs with this explanation by demonstrating it on the board. Here, the girls seem to have found an explanation by excluding this and possibly other single outcomes from the scope of applicability of the L1-pattern and therefore making the difference between short-term and long-term context explicit. Still, the construct of luck is only brought up in relation with the notion of the distribution of colours.

Episode 3: Building with the luck-construct for S-explanation for deviation

Over the course of all interviews, they again build with this construct to explain single outcomes of games being not in accordance with the theoretically expected pattern. One example is Episode 3 (about 35 min. later). So far, Ramona and Sarah have filled in several protocol sheets while playing more than 25 further games with a total of throws between 1 and 20 and have written down their strategy for betting. The interviewer's question leads Ramona to clarify the distinction between pattern and luck further:

1203 Interviewer Could you read out loud what you have written, Sarah?
1204 Sarah Always stay on the ant-
...
1206 Sarah As it has the most faces on the die and therefore you roll it more often.
1207 Interviewer Hm, you put that very well. What I don't get completely yet: I bet on the hedgehog and won, for example. Or – well, not only ant has won-
1208 Ramona That is just luck.
1209 Interviewer It's only luck?
1210 Ramona It is not a strategy, it is truly luck.

Here, the previously constructed S-explanation for deviation is *recognized* as being usable in a situation, in which the interviewer seems to point to single games. By emphasising the difference between luck and strategy, Ramona *builds with* it by referring it to the unpredictability of the single (lucky) outcomes in single short games (S-Prediction) and the more predictable L1-pattern (L1-predictability). This contributes to refining the distinction of S- and L1-context.

Episode 4: Constructing the L1-L2-distinction

In the second interview, Ramona and Sarah start to focus on the long-term context L2 of games with high totals of throws, which is supported by using the computer simulation and protocol sheets that include total of throws up to 1000. Ramona and Sarah address the question, when the red ant is a good bet without an interviewer's stimulus. Having filled in a protocol sheet and a series of 16 games with increasing totals of throws, they realize that their consequent bet on ant has won the first game, lost for the next four and won every game from the sixth one on (with totals of throws of 10, 100 and 1000):

975 Sarah *(points to sixth game on the protocol sheet; total of throws: 10)* From here on, you only always win with the ant.

This utterance could be an indication that she is constructing a notion of the predictability of the pattern ant-winning in relation to the context (distinguishing L2-predictability from L1-predictability). Although not marking exactly those games with at least 100 throws, her formulation "from here on" clearly addresses a series of games and seems vaguely to refer to the larger total of throws as they increase in the bottom of the sheet. While filling in a summary sheet, the girls become aware of their results showing clear patterns: If the total of throws was one or ten, all animals won, while the red ant was the only winner in all games with throw totals of 100 and 1000. Asked to formulate their strategy now, the following dialogue begins:

1079 Sarah At 100 and 1000, the ant always wins. At 10 and 1, it's always different-
...
1085 Ramona [At 10 and 1], mostly winning are-
1086 Sarah *(points to upper part of protocol sheet)* snail, frog and sometimes ant, too.

1087 Ramona There, ant is not winning as often and here *(points to lower part of protocol sheet)* you can see it, only ant.

The girls refine their construct of L2-pattern by contrasting it to the L1-pattern. Both seem to accept that ant is the only one winning in long games which is in accordance with Sarah's statement in 975. Referring to the series of short games (i.e. the L1-context), Sarah revises her previous statement and remarks in 1079, that at a throw total of one or ten "it's always different". It is possible that she emphasises the distinction between the L1 and L2 context and focuses the L1-deviation more than the L1-pattern itself. Furthermore, she might relate the absence of a pattern in L1 to the previous construct S-disturbance in the context of short games, which was then explained by the construct "luck". Here, both girls do not mention luck as a possible explanation, but point out the shift of context as the explanation for the discrepancies in the observations of patterns (L1-explanation). When Ramona starts to mention winning animals in 1085, Sarah points out three of four animals, relativising the red ant by adding the adverb "sometimes". In contrast to 1079, she is possibly now pointing out a L1-pattern, which is refined by Ramona in 1086. She emphasises the words "as often" and "always" and makes the distinction between her construct for L2-pattern (ant wins always) and L1-pattern (ant wins sometimes) very explicit. Furthermore, she hereby constructs the notion of both L1-deviation and L2-deviation.

CONCLUSION

Like the girls' pathway of developing conceptions, all ten interview-pairs create complex networks of constructs while trying to make sense of several, partially-conflicting experiences. In each case, a shift of focus between short-term and long-term context can be reconstructed. Beyond that, the individual pathways are highly individualised.

Ramona and Sarah are able to enrich pattern constructs with explanations not only in a long-term perspective, but also refine these patterns regarding the absence of patterns in a series and single outliers in short-term contexts. Their individual constructs "luck" and "pattern" seem not only to be connected to each other, but also to the distribution of colours. The deviation of patterns is only mentioned in relation with "pattern" and while mentioning explicitly the total of throws as being low. Though the girls don't compare this whole network of constructs and test its coherence, it seems from an outside point of view that by defining the scopes of applicability, their constructs are not contradictory, but in coexistence with each other. This gives evidence to the horizontal view on conceptual change and provides a short but deep insight into how the individual pathways of students can lead into conceptions consisting of networks of constructs, in which even rather idiosyncratic constructs such as luck have a scope of applicability that does not seem to obstruct the intended mathematical constructs. For some students, the negotiation of ranges of applicability of constructs is more complicated than for Ramona and Sarah. Further steps of data analysis include the identification of common conceptions for many participants and sharpen the description of the character of the network of constructs.

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