TOWARD AN INFERENTIAL APPROACH ANALYZING CONCEPT FORMATION AND LANGUAGE PROCESSES

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This paper introduces a theoretical approach to study individual conceptual development in mathematics classroom. It uses the theory of a normative pragmatics as an epistemological framework, which Robert BRANDOM made explicit in 1994. There are different levels of research in mathematics education on which BRANDOM's framework offers a consistent theoretical approach for describing such developments: a linguistic perspective, the theory of conceptual change and the theory of conceptual fields. Using that framework, we will outline an empirical example to describe technical language developments as well as developments of conceptual fields and of the students' conceptualizations.

INTRODUCTION

Many results of large-scale studies monitoring the education system (PRENZEL et al. 2007, ARTELT et al. 2000, BAUMERT et al. 1997, BAUMERT et al 1998) show for mathematics education that German students have difficulties with tasks that challenge their conceptual understanding. These difficulties seem to be caused by the German classroom practices, which do not challenge enough the students' individual cognitive skills, which lack teachers' diagnosis abilities, and which do not offer enough room for creative and individual work (e.g. PRENZEL et al. 2004).

Research is required in both mathematical learning environments and in formation of concepts and conceptualizations in order to find out in how far (i) the use of the specific potential which certain tasks offer and (ii) the dealing with students' conceptualizations have an effect on the formation of conceptual thinking. In Germany, there are only some studies which focus on the analysis of individual concept-formation (HUßMANN 2006, BARZEL 2006, HAHN / PREDIGER 2008, Prediger 2008a/b). There is also a demand for research with regard to dealing with certain individual students' conceptualizations.

Because mathematical thinking is genuinely conceptual thinking, the formation of mathematical concepts has gained big interest in the mathematics education research community. The multiple approaches and theories for describing and explaining conceptual processes and developments differ a lot in terms of their theoretical framework, e.g. developmental psychology or cognitive psychology. In this study, we choose a social-constructivist approach (COBB, YACKEL 1996).

With his theory of inferntialism, the philosopher Robert BRANDOM (1994) has introduced a convincing, comprehensive and coherent theoretical framework to analyze such language processes.

EPISTEMOLOGICAL FRAMEWORK: INFERENTIALISM

In his influential book on reasoning, representing and discursive commitment "Making it explicit" (1994) BRANDOM chooses an inferential approach to describe semantic content of concepts in terms of their use in practice: it is the idea that propositional semantic content can be understood in terms of the inferential relations they play in discourse, which means for example to know what follows from a proposition or what is incompatible with it. BRANDOM gives an analysis of discursive linguistic practice, describing a model of social practice -and especially a model of linguistic discursive practice- as a game of giving and asking for reasons, which means a normative pragmatics in terms of deontic scorekeeping. Using his theory to describe linguistic practice and based on the theory of a normative pragmatics introduced by BRANDOM (1994), we will develop an analytic tool to describe the formation of concepts. For BRANDOM, understanding

> "can be understood, not as the turning on of a Cartesian light, but as practical mastery of a certain kind of inferentially articulated *doing*: responding differentially according to the circumstances of proper application of a concept, and distinguishing the proper inferential consequences of such application." (BRANDOM 1994, p. 120)

In this sense, discourse can be described as a game of giving and asking for reasons, a term that can be traced back to WITTGENSTEIN'S 'Sprachspiel' (language game). Therefore, every 'player' in the game of giving and asking for reasons keeps score on the other players. This deontic score keeps track on the claims that every player (including oneself) is committed to and it keeps track on the commitments each one is entitled to. With every assertion –so with every move in the game of giving and asking for reasons- which one player is making, the score may change.

The inferential relations are commitment- and entitlement-preservations and incompatibilities. BRANDOM'S normative pragmatics gives an understanding of conceptual content on the basis of using the concepts in practice. "The aim is to be able to explain in deontic scorekeeping terms what is expressed by the use of representational vocabulary-what we are doing and saying when we talk about what we are talking *about*." (BRANDOM 1994, p. 496)

BRANDOM claims that the fact that propositions have a certain (propositional) content should be understood in terms of inferential relations. Accordingly, propositions are propositions because they have the characteristic feature to function as premises and conclusions in inferences (that means they function as reasons).

> "Thus grasping the semantic content expressed by the assertional utterance of a sentence requires being able to determine both what follows from the claim, given the further commitments the scorekeeper attributes to the assertor, and what follows from the claim, given the further commitments the scorekeeper undertakes. (…) In such a context, particular linguistic phenomena can no longer reliably be distinguished as 'pragmatic' or 'semantic'." (BRANDOM 1994, pp. 591/592)

It is important to note, that it is not necessary for an individual to know all the inferential roles of a certain concept to be regarded as someone that has conceptualized a certain concept. “To be in the game at all, one must make *enough* of the right moves-but how much is enough is quite flexible" (BRANDOM 1994, p. 636).

DERRY (2008) lines out the characteristics of an inferential view for education. Referring to BRANDOM and VYGOTSKY she notes that the

> “priorisation of inference over reference entails, in terms of pedagogy, that the grasping of a concept (knowing) requires committing to the inferences implicit in its use in a social practice (…). Effective teaching involves providing the opportunity for learners to operate with a concept in the space of reasons within which it falls and by which its meaning is constituted.” (DERRY 2008, p. 58)

CONCEPTUAL DEVELOPMENT RESEARCH IN MATHEMATICS EDUCATION

Using Robert BRANDOM’s ideas of a normative pragmatics, it is the aim of the project to develop a coherent theoretical framework within which the formation of concepts in mathematics education can be described. This theoretical framework uses inferential (instead of representational) vocabulary. There are different levels of research in mathematics education on which BRANDOM’s framework offers a consistent theoretical approach for describing such developments.

Theory of conceptual fields

Using Robert BRANDOM’s theory of a normative pragmatics as an epistemological background to describe formations of concepts, VERGNAUD’S theory of conceptual fields offers a consistent framework within which long- and short-term conceptual developments can be analyzed. Within his framework, he gives respect to both, mathematical concepts and individual conceptualizations.

WITTENBERG says that mathematics is “thinking in concepts” (1963). What distinguishes us as human beings, is the fact that we are concept users (Brandom 1994). Accordingly, not only mathematics is thinking in concepts: everything obtains a conceptual meaning for us and concepts are the smallest unit of thinking and acting. This decisive linguistic perspective of conceptual understanding was pointed out by SELLARS: “grasping a concept is mastering the use of a word” (see BRANDOM 2002, p. 87). Accordingly, it is necessary to research concept formation, which means it is necessary to study the classroom discourse. For that, VERGNAUD (1996, 1997) offers a solid theoretical framework. With his theory of conceptual fields, VERGNAUD developed a theoretical framework which picks up BROUSSAU’S theory of didactical situations (1997) and which offers a tool to describe, to analyze and to understand both short- and long-term formations of concepts. For him, a conceptual field refers to a set of (problem) situations, conventional and individual concepts.

> "(A) *conceptual field* is a set of situations, the mastering of which requires several interconnected concepts. It is at the same time a set of concepts, with different properties, the meaning of which is drawn from this variety of situations." (VERGNAUD 1996, p. 225)
>
> "A concept is a three-tuple of three sets: C = (S,I,S) where S is the set of situations that make it meaningful, I is the set of operational invariants contained in the schemes developed to deal with these situations, and S is the set of symbolic representations (natural language, diagrams (…)) that can be used to represent the relationships involved, communicate about them, and help us master the situations." (VERGNAUD 1996, p. 238)

In the latter definition, VERGANUD points out that language is essential for focusing on conceptual fields. Language is the surface on which we analyze formations of concepts. Conceptual fields are equally related to situations, to mathematical concepts, to individual conceptualizations and to operational invariants such as *theorems-in-action* or *concepts-in-action*. On the one side, those operational invariants are *theorems-in-action* which are "held to be true by the individual subject for a certain range of the situation variables" (VERGNAUD 1996, p. 225). On the other side, they are categories- or *concepts-in-action*,

> "that enable the subject to cut the real world into distinct elements and aspects, and pick up the most adequate selection of information according to the situation and scheme involved. Concepts-in-action are, of course, indispensable for theorems-in-action to exist, but they are not theorems by themselves. They cannot be true or false" (VERGNAUD 1996, p. 225).

In every new situation, the individual schemes develop. Because of the strong connection between situation and scheme, the short-term perspective on concept formation is important to study. At the same time, because of the individual development within the learning process and the different situations the individual deals with, the long term perspective is equally important to study.

Linguistic approach

Besides the theory of conceptual fields, there is a specific linguistic approach that can be drawn from BRANDOM'S epistemological framework. Therefore, SIEBEL (2005) refers to developments from colloquial to technical language by making implicit concepts explicit.

Thought and language is not the same, otherwise we would not be able to form sentences like "I don't know how to say it" or "that is not what I meant". Still, we can only get a precise picture of conceptual developments by observing the use of language, the discourse, that what's made explicit. To get an idea of what is implicit in use, we have to ask for reasons and commitments.

In her linguistic approach categorizing and analyzing technical language used in elementary algebra books, Siebel (2005) picks up that distinction. She distinguishes between explicit and implicit technical terms. Explicit ones are explicitly defined, e.g. by "x is called variable". Explicit technical terms are characteristic for explicit knowledge ('know-that') which can be made explicit in either words or formulas. In contrast, the meaning of implicit terms is characterized by their use (SIEBEL 2005, p. 120). Implicit technical terms are characteristic for implicit knowledge ('know-how') which can only be learnt by practical exercising. SIEBEL points out that most of our concepts are implicit and that we can only make some of them explicit (see SIEBEL 2005, p. 122). Referring to BREGER (1990), SIEBEL describes how knowledge and concepts develop from "know-how"- to "know that"-knowledge, from implicit to explicit knowledge– by making them explicit (2005, p. 122). That linguistic approach offers a description of developments from colloquial to technical language, lining out how implicit concepts and knowledge ("know-how") become explicit ("know-that").

Judgments as basic units

Following BRANDOM, the linguistic perspective cannot be separated from the propositional content. With every commitment and every judgment, we have taken on a certain kind of responsibility and committed ourselves to some explanation of the given phenomenon. Those explanations and judgments correspond to the theoretical schemes (see VERGNAUD 1996) which are intimately interwoven with the specific situation.

Theory of conceptual change

Following BRANDOM and VERGNAUD, learning and formation of concepts is closely linked to a specific situation. The developments that proceed in these situations are closely connected to the conceptualizations we have. These conceptualizations maybe have to be revised, expanded or modified in every new situation which we have to commit ourselves to, for example to a certain scheme or an explanation. The theory of conceptual change (e.g. DUIT 1996) picks up that distinction between individual conceptualizations and scientific conceptions.

The conceptual change theory is a constructivist approach to describe learning processes in terms of reorganization of knowledge (Duit 1996, p. 158, Prediger 2008b for an example in mathematics education). That means for the students to learn that their preinstructional concepts do not give sufficient orientation in certain scientific situations and for them to activate scientific conceptions in those situations (see DUIT 1996, p. 146). Learning scientific concepts often leads to conflicts with prior knowledge and familiar everyday concepts because certain features of both – familiar and new scientific concepts - seem to be incompatible. FISCHER and AUFSCHNAITER (1993) for example studied developments of meaning during physics instruction, focusing on the terms charge, voltage and field. Against the background of different levels of perception, they describe how the use of certain words changes during the learning process: "For this reason, at the beginning of the development of a subjective domain of experience it might be possible that words, as properties of objects, are not yet generated." (p. 165)

Summary

In all the perspectives above, there is a similar line of thought concerning the analysis and description of conceptual developments: intuitive concepts-in-action to consolidated mathematical concepts, implicit meaning of use to explicit technical language, preinstructional conceptualizations to scientific concepts. The aim of our project is to follow those lines among linguistic descriptions of expressions in mathematics classroom and to develop learning environments which considering the formation of concepts in mathematics classroom.

For this purpose, we study the development of individual long- and short-term conceptualizations and of formations of mathematical concepts within learning processes: what is the connection between (problem) situations and operational invariants (such as *theorems-in-action* or *concepts-in-action*)? What is the connection between the formation of concepts and symbolic expressions? In how far is it possible to classify the (problem) situations against the background of individual operational invariants?

Three aspects can be inferred from those questions: How does technical language develop? How do individual conceptualizations develop? How do conceptual fields develop? To examine these questions, we develop an empirical study to describe the individual learning processes.

ONE EXAMPLE ON (TECHNICAL) LANGUAGE DEVELOPMENT

To give an example of how the research questions outlined above can be approached, we offer some results of a small-scale study on technical language development (SCHACHT 2007). This example shows how technical language in chance-situations can develop, how individual conceptualizations develop and how the conceptual field of chance-situations has developed.

Short introduction to the study

For this purpose a fifth grade mathematics classroom of 30 students was observed and videotaped over a period of about six weeks. The central goal of the unit for the students was to develop a concept of ‘chance’. That means that in chance situations, the individual case will not be predictable, but focusing the long term, chance has a certain kind of mathematical structure (HEFENDEHL-HEBEKER 2003). Accordingly, one special focus of this unit was for the students to discover and experience the law of large numbers.

Main features of the unit concerning the research interests of the study were the focus on *discursive elements* in mathematics classroom, the focus on *reflection tasks* during the mathematical learning process and the focus on *student*- rather than teacher-*activity* (cf HUßMANN/PREDIGER 2009).

Based on a functional pragmatic approach, language was analyzed in terms of its use (e.g. EHLICH/REHBEIN 1986, KÜGELGEN 1994). The features of the unit mentioned above formed a solid base to analyze language developments of some students especially because they were often challenged to make their concept of chance explicit (either in written form or verbally).

Some results of the study

The results of this small scale study show some interesting phenomena which could be observed. We will outline one prototypical example of the study and describe its main features concerning technical language development as well as individual conceptualizations and conceptual fields development.

In this example, the task for the student Ralf was to describe and compare results of dice throws in different situations (10, 100, 500 and 1000 throws). Because of the qualitative differences of the situations which he is working in (description of absolute values → description of relative values), the technical language he uses leads to a paradox situation (distance (“Abstand”) is ‘small’ and ‘large’ at the same time). A couple of days after this situation, he uses a different and new term which seems more sufficient and viable.

More precisely, the student Ralf first uses the term ‘distance’ to compare some results of dice-throws. In the first scene, he uses the term ‘distance’ to compare absolute results.

Number	1	2	3	4	5	6
10 throws	3	1	0	4	0	2
1000 throws	165	174	169	161	171	160

Table 1: Similar example of dice results in absolute values (10 and 1000 throws)

Comparing results similar to Table 1, Ralf observes:

102 And in the situation with small numbers of throws
103 the distances (“Abstände”) get smaller.

There are two aspects to point out concerning the use of the term ‘distance’. First, he compares the dice results by noticing that the “distances get smaller” (line 103) the smaller the number of throws is. In the example above, that means that there is a little distance between the one time ‘2’ and the two times ‘6’ but there is a greater distance between the 160 times ‘6’ and the 171 times ‘5’. Second, he uses the term distance to distinguish between situations with a high number of throws (e.g. 1000 throws) and a small number of throws (100 throws).

Later in the same lesson, he uses the same term (‘distance’) again to compare dice results, except now they are given in relative values (in percent). The teacher asks the students to compare a couple of histograms which show the results of 10-100-500-1000 throws. The histograms which show results of 10 throws of course look quite different to those with 1000 throws. The latter ones show the stabilization of the relative distribution (law of large numbers) while the others show that the results with for example 10 throws differ quite a lot.

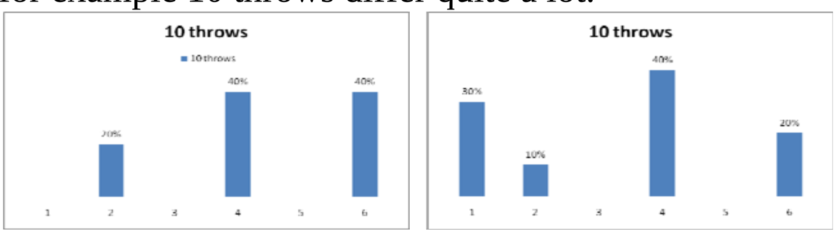
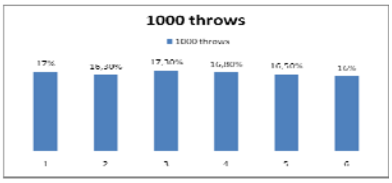


Table 2: Similar example of histograms showing dice results in relative values (10 throws)



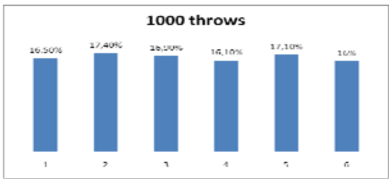


Table 3: Similar example of histograms showing dice results in relative values (1000 throws)

The teacher asks Ralf, what he noticed. Ralf answers:

8 Ralf: I observed that,
9 given a small number of throws,
10 the distances ("Abstände") become larger
11 and given a large number of throws,
12 the distances ("Abstände") become smaller.

In this situation Ralf describes that the distances become larger given a small number of throws. It seems plausible that he has a horizontal perspective and compares all histograms showing the results of 10 throws whereas the "distances become smaller" comparing the others with for example 1000 throws.

At the same time, like in the situation above, Ralf is using the term 'distance' again to distinguish the small and the large 'number-of-throws situations'. Except that he uses the term conversely: in the first situation he described the distances to become smaller when the number of throws becomes *smaller* (lines 102/103), in the latter situation he observes the distances to become larger when the number of throws becomes *smaller* (lines 8/9).

Comparing both examples, the difficulty is that the quality of the situation changes: in the first situation, Ralf compares the absolute values of the dice results of 100 and of 1000 throws. He recognizes that the distances of the results with 10 throws are smaller than the ones with 1000 throws (lines 102/103).

Accordingly, although the term 'distance' is a quite helpful and viable term *in each situation* to distinguish between small and high number of throws, it is overall not sufficient because it seems to lead to paradox and incompatible results.

Some days later the students are asked to give a written comment on the following sentence: "You cannot predict the result of throwing a single dice, but in the long run you don't have a random result." Ralf writes:

130 Given a small number of throws
131 you cannot predict
132 chance, but
133 given a higher number of throws, that works better
134 because it is more distributed ("verteilter") there.

The next day, he adds on a working sheet in a similar situation:

5* in the situation of thousand throws, the distribution ("Verteilung") is: (…)

In both quotes, Ralf uses the term 'distribution' / 'distributed' to distinguish between small and large numbers of throws. For him, this term works without inconsistencies to distinguish both situations. He is also able to predict a distribution in the large number-of-throws situation (line 5*).

Summary

Focusing on technical language development from a linguistic perspective, this example describes a development of the intuitive and implicit use of the term 'distance' to an explicit use of the technical term 'distribution' that is viable to distinguish between small and large number of throws.

There are two different concepts-in-action Ralf uses: in the first situation, he has a binary concept for comparing the results. In the other situation, Ralf observes a certain structure given a high number of throws. Here, his concept-in-action is that given a high number of throws and a certain mathematical structure, chance is predictable. That effects his theorem-in-action: given a high number of throws, the (relative) distribution can be predicted quite precisely.

This development shows his conceptual change regarding chance situations: whereas his intuitive conceptualization focuses on the term 'distance', he then is able to activate a mathematical conception on chance situation using the technical term 'distribution' which focuses on the long-term perspective on chance situations. The conceptual change is in line with the dynamic development of Ralf's theorem-in-action: the new problem situation leads him to come up with a new theorem-in-action.

This example shows in how far all three levels are connected in terms of the inferential epistemological approach that BRANDOM introduces: both conceptual change and conceptual fields help to observe the formation of concepts. But these processes can only be studied because we are concept users (BRANDOM): language is the surface on which the linguistic analysis of the formation of concepts operates.

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